

Your Signature _____

1. (15 points) Let (X, Y) be random variables with joint probability density function, $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ given by

$$f(x, y) = \begin{cases} K(y^2 - x^2)e^{-y} & \text{if } -y \leq x \leq y, 0 < y < \infty \\ 0 & \text{otherwise} \end{cases}$$

Find K and compute the marginal densities X and Y .

2. (10 points) Let the X_1 be the time taken by Asif to solve Quiz 1. Let X_2 be the time taken by Suppandi to solve Quiz 1. Assume that X_1 and X_2 are independent and Uniform $(0, 10)$ random variables. Find the probability that Asif will take at least four times as long as the Suppandi to solve the quiz.

3. (10 points) Let $\lambda > 0$. Let $X \sim \text{Gamma}(2, \lambda)$ and be independent of $Y \sim \text{Gamma}(1, \lambda)$. Set $Z = X + Y$. Find the conditional expectation of X given $Z = z$ for all $z > 0$.

4. (15 points) Let (X, Y) be a bivariate Normal random variable, with $\rho^2 \neq 1$. It is known that:

- $Y \sim \text{Normal}(0, 1)$ and
- for any $y \in \mathbb{R}$, $X | Y = y \sim \text{Normal}(9y + 4, 9)$.

Find the marginal density of X .